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Trend analyses with river sediment rating curves

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Abstract:

Sediment rating curves, which are fitted relationships between river discharge (Q) and suspended-sediment concentration (C), are commonly used to assess patterns and trends in river water quality. In many of these studies, it is assumed that rating curves have a power-law form (i.e. $C = aQ^b$, where a and b are fitted parameters). Two fundamental questions about the utility of these techniques are assessed in this paper: (i) how well to the parameters, a and b, characterize trends in the data, and (ii) are trends in rating curves diagnostic of changes to river water or sediment discharge? As noted in previous research, the offset parameter, a, is not an independent variable for most rivers but rather strongly dependent on b and Q. Here, it is shown that a is a poor metric for trends in the vertical offset of a rating curve, and a new parameter, $\hat{a}$, as determined by the discharge-normalized power function $[C = \hat{a}\ (Q/Q_{GM})^b]$, where Q_{GM} is the geometric mean of the Q-values sampled, provides a better characterization of trends. However, these techniques must be applied carefully, because curvature in the relationship between log(Q) and log(C), which exists for many rivers, can produce false trends in $\hat{a}$ and b. Also, it is shown that trends in $\hat{a}$ and b are not uniquely diagnostic of river water or sediment supply conditions. For example, an increase in $\hat{a}$ can be caused by an increase in sediment supply, a decrease in water supply or a combination of these conditions. Large changes in water and sediment supplies can occur without any change in the parameters, $\hat{a}$ and b. Thus, trend analyses using sediment rating curves must include additional assessments of the time-dependent rates and trends of river water, sediment concentrations and sediment discharge. Published 2014. This article is a U.S. Government work and is in the public domain in the USA. *Hydrological Processes* published by John Wiley & Sons Ltd.

KEY WORDS suspended-sediment discharge; sediment rating curve; trend analyses

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INTRODUCTION

Perhaps the most fundamental and important information to track the status and trends of river water quality are records of discharge and suspended-sediment concentration that are collected by government resource agencies, academic institutions and private citizens (Milliman and Farnsworth, 2011). In combination, these records of discharge and suspended-sediment concentration can provide insights to the patterns and variability of river sediment discharge in time and space (Meade *et al*., 1990; Meade, 1994; Asselman, 1999; Hicks *et al*., 2000; Walling and Fang, 2003) and the effects of disturbances such as wildfire, earthquakes, torrential rainfall, landslides, land use change, and dam construction and removal (e.g. Dadson *et al*., 2004; Syvitski *et al*., 2005; Hicks and Basher, 2008; Kao and Milliman, 2008; Wang *et al*., 2008; Horowitz, 2009; Meade and Moody, 2010; Huang and Montgomery, 2013; Curran *et al*., 2014). Thus, multiyear to decadal sets of river suspended-sediment concentrations are valuable because they can reveal trends related to the effects of natural and human-caused changes to watersheds (Figure 1).

To characterize patterns and trends in river sediment concentrations, data are often fit with regression techniques, such as the commonly used linear regression between log-transformed discharge (Q) and suspended-sediment concentration (C) data:

$$\log(C) = b\log(Q) + \log(a) \tag{1}$$

where a and b are the rating parameters found from regression (Syvitski *et al*., 2000). In this technique, b is the slope between the log-transformed Q and C data, and log(a) is the ‘y-intercept’ or ‘offset’ value of log(C) defined where log(Q) is equal to zero (i.e. where Q is equivalent to 1 in the units of discharge, which are most commonly m^3/s). Note that Equation (1) is often reported in the power-law formulation:

$$C = aQ^b \tag{2}$$

A global analysis suggested that a and b are related to river basin characteristics such as topographic relief and

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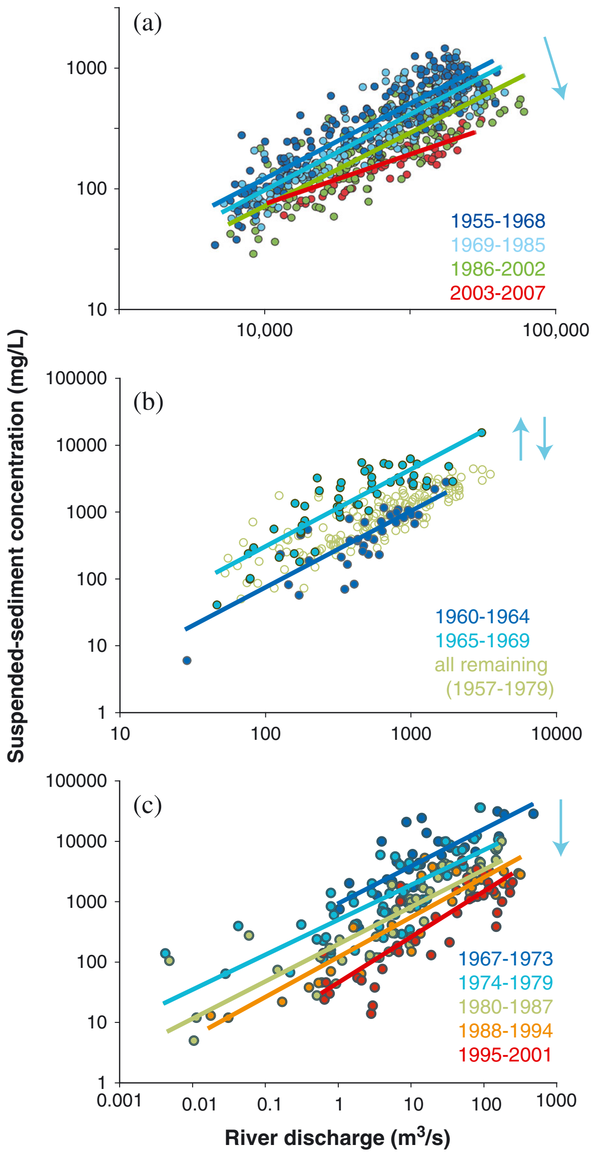

Figure 1. Suspended-sediment concentrations from three rivers that reveal strong time-dependence in the discharge-concentration relationships. (a) Lower reach of the Changjiang (Yangtze River) at Datong, watershed area = 1 800 000 km^2, after Hu *et al.* (2011), (b) Trinity River at Hoopa, California, USGS Station 11 530 000, watershed area = 7390 km^2, after Warrick *et al.* (2013), and (c) Santa Ana River, California, USGS Station 11 078 000, watershed area = 4400 km2, after Warrick and Rubin (2007). Lines are power functions fit through data subsets from the water years noted. Arrows show temporal trends in the rating curves

runoff (Syvitski *et al.*, 2000), although they can also vary significantly in time owing to sediment availability in the watershed and other factors (Asselman, 1999; Dadson *et al.*, 2004; Yang *et al.*, 2007; Huang and Montgomery, 2013; Figure 1).

Equations (1) and (2) are often referred to as 'sediment rating curves' owing to their potential usefulness for predicting suspended-sediment concentrations for intervals of time without samples (Asselman, 2000). Predictions of continuous suspended-sediment concentrations are valuable because these records allow for calculation of river sediment discharge from the product of continuous sediment concentration, water discharge and log-transform bias correction, if appropriate (Ferguson, 1986; Cohn *et al.*, 1992). The utility of suspended-sediment rating curves in these calculations varies from river to river and is largely a product of the patterns of supply and transport of suspended-sediment over multiple timescales (Asselman, 1999; Horowitz, 2003; Aulenbach and Hooper, 2006). As such, there have been disparate conclusions about the utility of sediment rating curves for calculating river sediment loads (e.g. Walling, 1977; Horowitz, 2003; Wright *et al.*, 2010).

Changes in sediment rating curve parameters, *a* and *b*, over time have been noted for many rivers systems (e.g. Syvitski *et al.*, 2000; Warrick and Rubin, 2007; Yang *et al.*, 2007; Huang and Montgomery, 2013), and there is a general assumption that these changes reflect alteration of the erodibility and/or supply of sediment in the watershed, the power of the river to erode and transport sediment or the spatial scale of the basin (Asselman, 2000). Owing to the widespread alteration and development of the global land surface (Goldewijk *et al.*, 2011; Lambin and Meyfroidt, 2011), hydrologic changes that may result from human-caused alterations of the global climate system (Vörösmarty and Sahagian, 2000; Milly *et al.*, 2008; Elsner *et al.*, 2010), the need to restore river sediment supplies as part of ecosystem resilience and recovery plans (Kirwan *et al.*, 2010; Duda *et al.*, 2011), and the inherent variability of river sediment transport even without these human pressures, it is likely that river sediment concentrations and the associated sediment rating parameters will continue to change in the future. It is, therefore, important to have adequate techniques to quantify and characterize changes to these river systems.

Many types of changes in sediment rating curves are possible, including the alteration of the vertical offset, the slope or both variables (Figure 2). Although the examples in Figure 1 show changes dominated by the vertical offset, there are numerous examples of time-dependent changes in rating curve slope (e.g. Warrick and Rubin, 2007; Wang *et al.*, 2008) a few of which will be highlighted later in this paper. Although trends in sediment rating curves are intriguing, it is arguably more important to determine the causes of change. For example, changes in the vertical offset of a rating curve (Figure 2d and f) may be associated with changes in watershed sediment production (e.g. Wang *et al.*, 2008; Warrick *et al.*, 2012), or – as shown by Warrick and Rubin (2007) for the Santa Ana River of California (*cf.* Figure 1c) – associated with increases in river discharge that, in effect, dilute the relatively steady suspended-sediment supplies with time. These contrasting results show the need for a better synthesis of the effects of

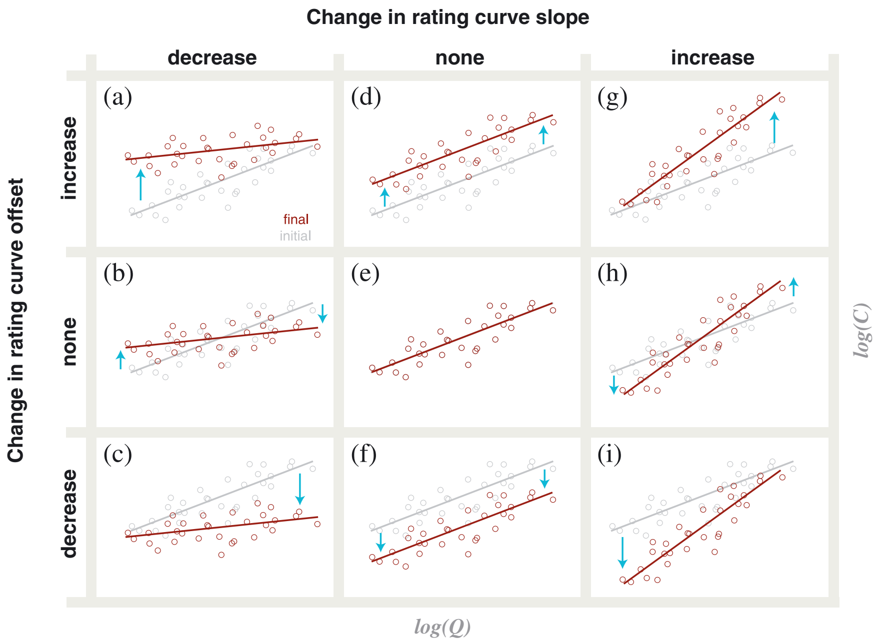

Figure 2. Conceptual trends in the relationships between river discharge (Q) and suspended-sediment concentration (C) as expressed by changes in the sediment rating curve slope and vertical offset. Each subplot shows log-transformed synthetic Q and C data from two time intervals, initial (light color) and final (dark color). Arrows show the trends in the least-squares power function rating curves (lines)

sediment and water supply on sediment rating curve parameters, so that future investigations can better assess the causes of sediment rating curve changes.

The goal of this paper is to evaluate the utility of sediment rating curves for measuring and determining water quality trends in rivers. Two questions about the utility of sediment rating curves are assessed: (i) how well to the parameters, a and b, characterize trends in the data, and (ii) are trends in rating curves diagnostic of changes to river water or sediment discharge? These questions are addressed by exploring the mathematical implications of Equations (1) and (2), investigating empirical data from a range of river systems and simulating the effects of simple river water and sediment discharge changes using Monte Carlo techniques. As such, this paper does not attempt to supersede the synthesis of statistical techniques and trend analyses provided by Helsel and Hirsch (2002); rather, the work here provides narrow focus upon the use and utility of sediment rating curves for trend analyses. In fact, as noted in the Discussion and Conclusion Sections, there are an additional techniques suggested by Helsel and Hirsch (2002), Aulenbach and Hooper (2006), Horowitz (2009), Hirsch *et al.* (2010) and several others that can provide better sediment discharge and trend analyses than those provided by simple rating curve analyses. Furthermore, it is concluded that all sediment rating curve analyses require additional assessments of river water and sediment discharge trends. Yet, sediment rating curves are popular, easy to understand and – if used properly – useful. Thus, it is important to better describe their utility.

EXAMINATION OF RATING EQUATIONS

Consider the rating parameters, a and b, that are generated from a least squares regression through Q and C data using Equation (1). The offset parameter, a, has meaningless units that are equivalent to $M\ T^{b}\ L^{-(1+3b)}$, or for the International System of Units (SI): kg s^{b} $m^{-(1+3b)}$. That noted, it can be shown that the magnitude of a is equivalent to the suspended-sediment concentration at Q of 1 m^3/s, which is the condition for which log(Q) is zero for SI (Syvitski *et al.*, 2000).

The choice of units for Q is important for several reasons. First, a is dependent upon the discharge units (Syvitski *et al.*, 2000). Readers can witness this effect if two separate regressions are conducted with the same Q and C data: one with discharge in m^3/s and the other with discharge converted to ft^3/s. If this is carried out, note that the b-values will be equivalent, but the log(a) values will differ by a factor of $b*\Delta\log(Q)$, where $\Delta\log(Q)$ is equivalent to the difference of the log-transformed Q-values for the two y-intercepts after they are transferred into similar units. Thus, for the present example, $\Delta\log(Q)$ is equivalent to log(1) – log(0.02832) or 1.55. Thus, $\Delta\log(a)$ will be equal to $1.55*b$ for this example. This simple exercise shows how a-values are dependent on both the units of Q and the values of b. The units of discharge for a study often reflect the spatial scale of the study (plot *vs* watershed) and organization conducting the study, and thus, several discharge units are found in the literature: m^3/s, ft^3/s, L/s, mm/s (normalized to drainage area) and others.

Second, a is commonly meaningless for most rivers because it is never realized. For example, imagine the regression lines for the Changjiang shown in Figure 1a extended down to discharge values of 1 m^3/s. Data provided by Hu *et al*. (2011) suggest that these a-values range between 0.0009 and 0.04 mg/l, which are many orders of magnitude lower than the reported long-term mean concentrations of 160–460 mg/l and lower than the ~30 mg/l minimum concentration measured for this river during five decades of monitoring (Figure 1a). Other examples abound. For example, discharge in the Mississippi River is commonly five orders of magnitude greater than 1 m^3/s (Meade and Moody, 2010). How, then, should one interpret and compare values of a if suspended-sediment concentrations never approach these values? The best answer to this question is that it is not possible to provide a relevant physical meaning for a under these conditions (*cf*. Asselman, 2000).

Third, and most important for the trend analyses evaluated here, changes in the offset parameter, a, between two intervals of time may not reflect the actual vertical offsets between rating curves. This can be shown graphically by considering a rating curve that increases in slope without change to the mean vertical offset (Figure 3a). This scenario is equivalent to a simple 'rotation' of the log(Q) and log(C) data by Δb (Figure 3b). For this example, log(Q) = 0 is less than the sampled values of log(Q) much like the examples discussed earlier. Under this scenario, the change in the offset parameter, Δlog(a), is negative in contrast to the negligible vertical offset found in the data (Figure 3b). That is, b and a are directly and inversely related (Figure 3c), which is a pattern expressed in many rivers (Asselman, 2000; Syvitski *et al*., 2000). Thus, it can be concluded that Δlog(a) does not provide an accurate measurement of the change in vertical offsets of most sediment rating curves.

Another graphical example is valuable to emphasize this. Consider a sediment rating curve that exhibits increases in both slope and vertical offset such that the final rating curve plots entirely above the initial curve (Figure 4a). If the location of log(Q) = 0 is not near the center of the distribution of sampled log(Q) values, then Δlog(a) may suggest a very different trend than exhibited in the data. For the condition shown in Figure 4b, Δlog(a) will be negative, which is in the opposite direction of the positive vertical offset the between curves.

Thus, a better measurement of the vertical offset between two sediment rating curves will be obtained near the centre of the log(Q) distribution. An ideal way to do this is to conduct the regression with discharge-normalized data:

$$\log(C) = b\log\left(\frac{Q}{Q_{GM}}\right) + \log(\hat{a}) \tag{3}$$

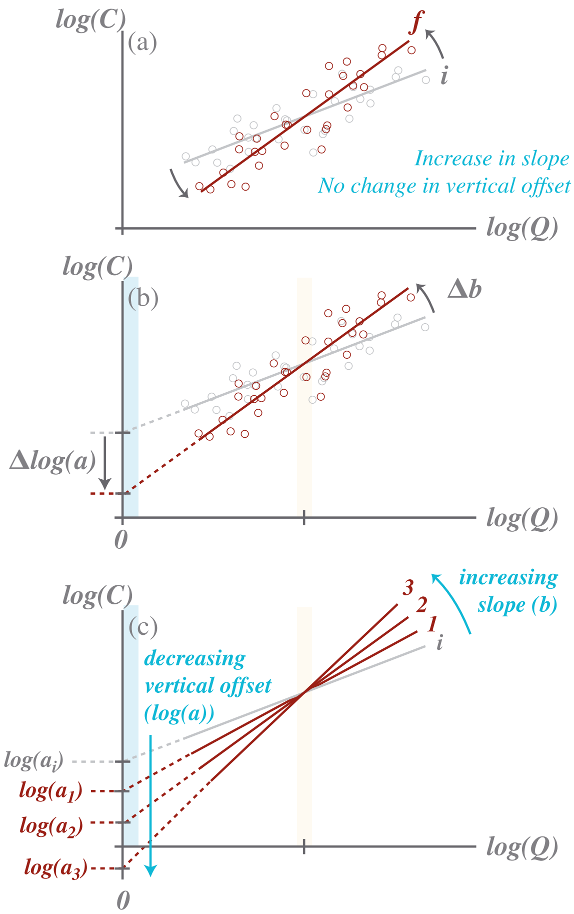

Figure 3. An example of how changes in the relationship between discharge (Q) and suspended-sediment concentrations (C) influence sediment rating curve parameters. (a) A condition in which the rating curve slope increases without a change in the vertical offset, thereby resulting in a rating curve rotation between the initial (i) and final (f) conditions. (b) Linear regressions through the log-transformed data (lines) reveal that Δlog(a) is nonzero when the location of log(Q) = 0 is not near the center of majority of samples. (c) Under these conditions, the parameters b and log(a) are inversely related, even though the data exhibit no change in vertical position. Thus, a better measure of the 'vertical offset' in the data is found at the center of the distribution of log(Q), which is highlighted with tan vertical bars

$$C = \hat{a}\left(\frac{Q}{Q_{GM}}\right)^{b} \tag{4}$$

where $\hat{a}$ is a vertical offset parameter (kg/m^3 or mg/l) equivalent to the suspended-sediment concentration of the middle of the sample distribution and Q_{GM} is the geometric mean of the Q data (Figure 4c). The geometric mean is the optimal normalization parameter because it is the center of mass of the log(Q) data and because the least squares technique uses the mean of the log(Q) to compute the slope and offset parameters. Applying Equation (3) to the present example reveals that Δlog($\hat{a}$) is positive, which is consistent to the vertical offset between the curves (Figure 4c). Also note that $\hat{a}$ has meaningful units of

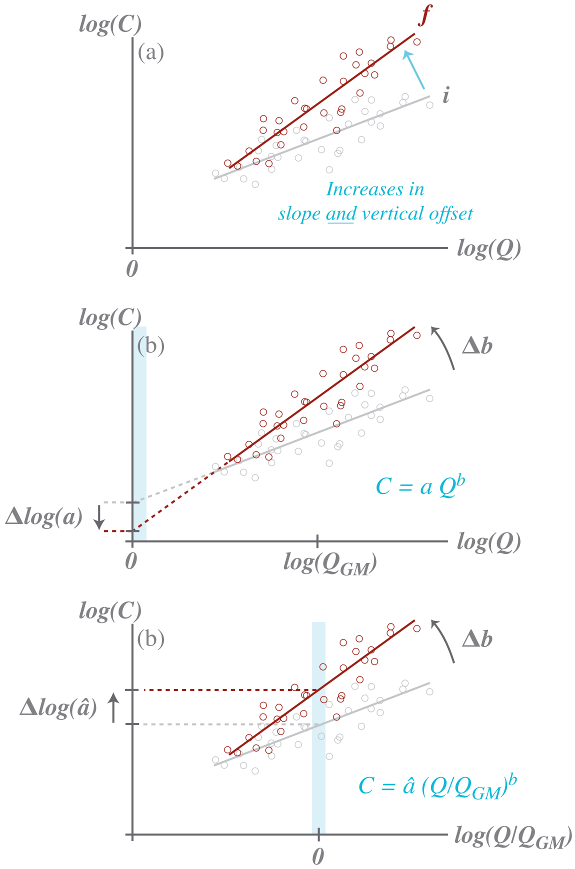

Figure 4. Comparison of two techniques to calculate the change in the rating curve vertical offsets under the situation that both slope and vertical offset increase between the initial (*i*) and final (*f*) conditions, and $\log(Q) = 0$ is not in the center of the $\log(Q)$ data as shown in (a). (b) Using the standard rating equation, $\Delta\log(a)$ is computed to be negative, which is counter to the increase in vertical offset. (c) Using the rating equation proposed here, in which discharge data (Q) are normalized by the geometric mean of the discharge samples (Q_{GM}), change in the offset parameter ($\Delta\log(\hat{a})$) better characterizes the vertical offset between the two sets of data

concentration, unlike a. Note, however, that a trend analysis must use a uniform value of Q_{GM} for all time intervals. If different Q_{GM} are used in an analysis, the calculated $\hat{a}$-values will be biassed by the range of Q_{GM}-values used.

Combining Equations (1) and (3), it can be shown that a and $\hat{a}$ are related by:

$$\log(a) = \log(\hat{a}) - b\log(Q_{GM}) \quad (5)$$

This suggests that a and $\hat{a}$ will be equal if $\log(Q_{GM}) = 0$, that is, if $Q_{GM} = 1\ \mathrm{m^3/s}$ for SI. Otherwise, a will be dependent upon the vertical offset of the center of the curve ($\hat{a}$) and the values of b and Q_{GM}. Because b are almost universally positive (Syvitski *et al.*, 2000), $\log(a)$ will be smaller than $\log(\hat{a})$ for all Q_{GM} greater than $1\ \mathrm{m^3/s}$.

Thus, it can be concluded that a-values do not provide independent measurements of the changes in vertical offsets of sediment rating curves but rather are largely dependent on b and Q_{GM}. This can be further illuminated with investigation of the annual sediment rating curve parameters for the Changjiang (Yangtze River) at Luoshan calculated and tabulated by Yang *et al.* (2007). These data are unique and valuable, especially to explore the implications of Equations (1)–(5). Time series of the reported annual values of $\log(a)$ and b for this station are shown in Figure 5a and b. Both variables reveal significant ($p < 0.0001$) time-dependent, but inversely related, trends over the three-decade record. The effect of the increase in b and decrease in $\log(a)$ over time on the annual sediment rating curves is shown in Figure 6. It is clear from Figure 6 that the slopes of the curves increased over time, consistent with Figure 5a. However, Figure 6 reveals that there are only negligible to moderate changes in the vertical offset between the curves, which contrasts with the three order-of-magnitude decrease in a-values suggested by the $\log(a)$ data (Figure 5b).

Application of Equation (4) to the Changjianag data using an assumed value of $20\,000\ \mathrm{m^3/s}$ for Q_{GM}, results in the same time-dependent patterns in b (Figure 5d) but only moderate increases in $\log(\hat{a})$ over the record (Figure 5e). It is notable that the trends in $\log(a)$ and $\log(\hat{a})$ are in the opposite direction (Figure 5), which is consistent with the implications of Equation (5) for Q_{GM} greater than $1\ \mathrm{m^3/s}$. Also, $\log(\hat{a})$ has low dependence on b, (linear $r^2 = 0.10$; Figure 5f), which supports the concept that it is a more independent measurement of the vertical offset than a.

In conclusion, although the standard rating curve equations (Equations (1) and (2)) are mathematically equivalent to the proposed discharge-normalized equations (Equations (3) and (4)), they do not provide the same metrics for analysing trends in sediment rating curves. The discharge-normalized equations proposed here (Equations (3) and (4)) provide independent assessments of both the slope and vertical offset of sediment rating curves, which make them superior to the standard equations.

A NOTE ABOUT CURVY DATA

Until this point, it has been assumed that the power-law relationships of Equations (1)–(4) represent the linear patterns between $\log(Q)$ and $\log(C)$ data well. However, as noted generally for river constituents by Helsel and Hirsch (2002) and specifically for suspended-sediment concentrations by work such as Hicks *et al.* (2000), there may be significant curvature (or nonlinearity) in the $\log(Q)$ and $\log(C)$ relationship such that Equations (1)–(4) do not adequately characterize the underlying patterns. Under

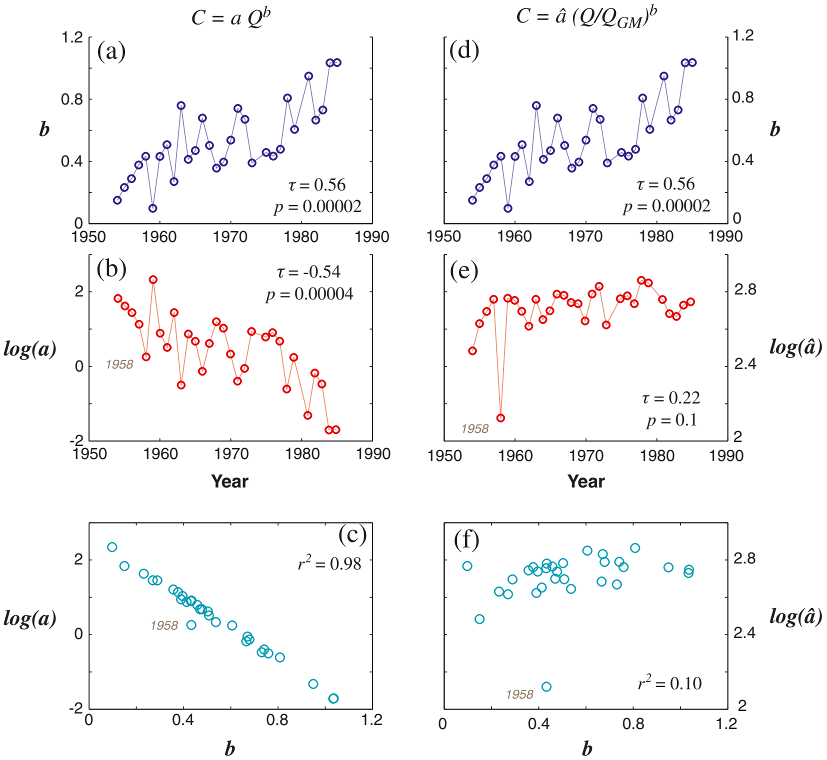

Figure 5. Annual sediment rating curve parameters from the Changjiang (Yangtze River) at the Luoshan gaging station from data tabulated in Yang *et al.* (2007). (a–c) Rating parameters computed and tabulated by standard techniques by Yang *et al.* (2008). (d–f) Rating parameters computed with the discharge-normalized formulation of Equation (3). The nonparametric Kendall tau (τ) and *p*-values are provided for the time-series and reveal whether temporal trends in the data are positive or negative ($\tau > 0$ and $\tau < 0$, respectively). Linear correlation coefficients (r^2) are provided for the scatter plots of *b* versus log(*a*) and log(*â*). Results from 1958 are highlighted as different from the remaining years

these circumstances, Helsel and Hirsch (2002, p. 336) recommended using the locally weighted scatter smoothing (lowess) fitting technique of Cleveland (1979) and trend analyses using residuals about these functions.

The effects of curvature in the log(Q) and log(C) relationship on sediment rating curves and trend analyses derived from these curves can be profound. For example,

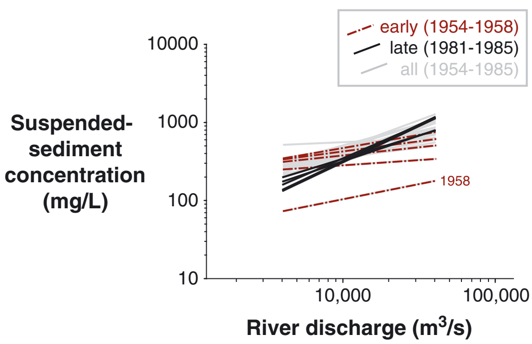

Figure 6. Annual sediment rating curves from the Changjiang (Yangtze River) at the Luoshan gaging station (after Yang *et al.*, 2007) highlighting the changes that have occurred during 1954–1985. Curves are plotted over the same range of discharge values, 5000–40 000 m^3/s, for comparative purposes

consider suspended-sediment data from the Eel River at Scotia (USGS Station 11477000; Figure 7), which have both curvature between log(Q) and log(C) and time-dependent trends during the 1955–1998 sampling record (*cf.* Williams, 1989; Warrick *et al.*, 2013). As shown with a lowess fit through the log(Q) and log(C) data, the slope between these variables is greatest between approximately 50 and 500 m^3/s and lower for discharges values outside of this range (Figure 7a).

The nonlinear pattern in the Eel River log(Q) and log (C) data may influence sediment rating curves derived with Equations (1)–(4) if a limited subset of the data are assessed. For example, three water years with different ranges of sampled discharge (1974, higher flows; 1980, moderate flows; and 1981, lower flows) are compared in Figure 7b. The *b*-value for the 1980 data is twofold to threefold greater than those computed for 1974 and 1981 (2.1 *vs* 0.75–0.83; Figure 7b). This is consistent with the fact that the 1980 data were collected within the high-sloped region of the data and 1974 and 1981 were not. These three annual rating curves reveal differences in vertical offsets, too. For example, the somewhat parallel curves of 1974 and 1981 are about an order-of-magnitude apart in the vertical, or concentration, dimension if compared near the Q_{GM}-value of 400 m^3/s (Figure 7b).

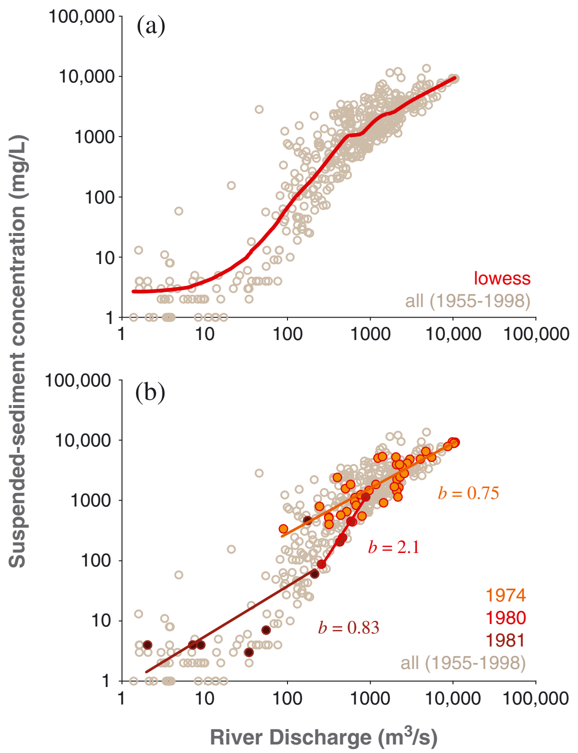

Figure 7. Suspended–sediment concentrations from the Eel River at Scotia (USGS Station 11 477 000, drainage area = 8063 km^2), which reveal the effects of curvature in the log-transformed discharge and concentration data on sediment rating curve parameters. (a) The complete set of available suspended-sediment data including a lowess fit after Warrick *et al.* (2013) that reveals curvature. (b) Subsets of the complete sample records from three water years (1974, 1980, and 1981) highlighting large differences in the fitted slopes (*b*) consistent with the range of discharge values sampled during each year

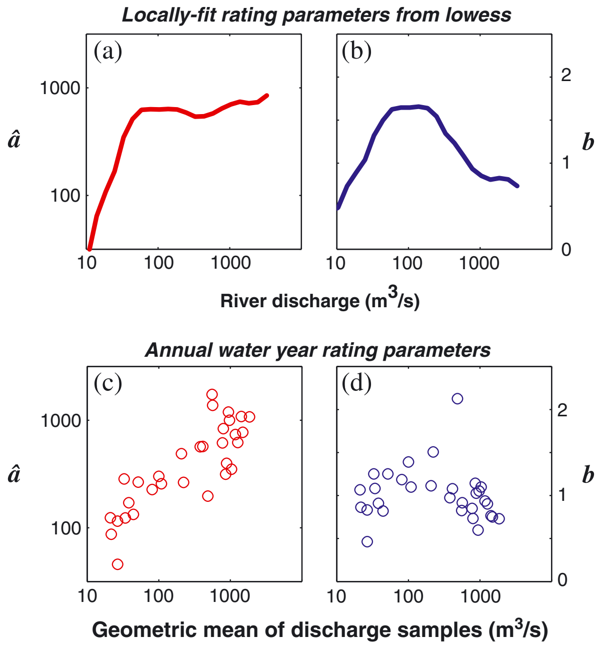

Figure 8. The effects of curved patterns between log(Q) and log(C) on the rating parameters ($\hat{a}$, b) calculated from subsets of the sample data. Results calculated for the Eel River at Scotia (USGS Station 11 477 000) that has discharge-dependent patterns in $\hat{a}$ and b (cf. Figure 7). (a,b) Locally fit rating parameters along the lowess curve shown in Figure 7(a) calculated using samples from the nearest ±0.5 of log(Q) discharge values. (c,d) Annual suspended-sediment rating parameters during 1956–1998 for the Eel River compared with the geometric mean discharge for the samples from each water year. Consistent patterns are shown for both â (increase with discharge) and b (maximum near 100 m^3/s)

These observations are consistent with a comparison of $\hat{a}$ and b from the lowess curve-fit and annual water year values of $\hat{a}$ and b, both of which have strong discharge dependence (Figure 8). The offset parameter, $\hat{a}$, increases with discharge (Figure 8a and c), whereas the slope parameter, b, is greatest near ~100 m^3/s (Figure 8b and d). Without curvature in the log(Q) and log(C) relationship, there should be very limited discharge dependence in $\hat{a}$ and b. The discharge dependence in $\hat{a}$ was not associated with differences in Q_{GM}-values, because a consistent Q_{GM}-value of 400 m^3/s was used for all calculations.

These discharge dependencies in $\hat{a}$ and b make it difficult to interpret time-dependent trends in the suspended-sediment concentration data. For example, values of $\hat{a}$ and/or b of the Eel River are directly related to the discharge values sampled, and these values are related to climatic or hydrologic shifts and/or the focus of sampling programs. Although beyond the scope of this paper, the Eel River data presented earlier provides evidence for both sediment supply changes and a shift in sampling strategy toward more frequent low-flow sampling (*cf.* Sommerfield *et al.*, 2002; Warrick *et al.*, 2013; Warrick, 2014). Thus, for rivers with curvature in the relationship between log(Q) and log(C), such as shown here for the Eel River, the power function rating curve formulas do not provide independent assessments of time-dependent patterns and trends in the suspended-sediment data. As noted in the Discussion and Conclusion Sections, better trend analysis techniques exist for these conditions, including analysis of residuals about the nonlinear lowess function.

THE EFFECTS OF WATER AND SEDIMENT DISCHARGE ON RATING PARAMETERS

Although it is important to evaluate whether rating curves change with time, it is arguably more important to determine the hydrologic processes causing these changes. Because the fundamental processes influencing sediment rating curves include runoff, erosion and water and sediment routing throughout a landscape and channel network (e.g. Asselman, 2000; Syvitski *et al.*, 2000), it is important to consider how these processes influence Q and C and thus the sediment rating curve. Perhaps the most obvious process that potentially influences a rating curve is the rate of sediment supply to a river (e.g. Yang *et al.*, 2007; Warrick *et al.*, 2012;

Huang and Montgomery, 2013). Because the majority of suspended sediment is transported as washload and is dependent on sediment supply rates to the river channel (Asselman, 2000; Syvitski *et al*., 2000), river suspended-sediment concentrations react strongly to increases or decreases in supplies of washload sediment. Another important, yet lesser understood variable is the rate of discharge in the river, which is related both to the ability of water to erode and carry suspended sediment and to the ability of river discharge (without a significant sediment source) to dilute existing suspended-sediment concentrations in a river channel. Warrick and Rubin (2007) provide an example of these dilution trends for the Santa Ana River of California (*cf*. Figure 1c), for which runoff from lowlands increased with time from urbanization, while sediment supplies from upland hillslopes remained relatively constant.

Although other variables exist that can alter patterns between Q and C such as the grain-size distributions of suspended and bed sediment (Walling, 1974; Rubin and Topping, 2001), the following analysis will rely on two simple parameters – water and sediment discharge – as controlling factors for sediment rating curves. This is not to dismiss other important processes but rather to highlight the strong results obtained from these two parameters.

Unfortunately, there are no river sampling records known that include adequate histories of both increasing and decreasing functions of water and sediment discharge with time. Thus, synthetic records were generated on the basis of simplified hydrologic properties and Monte Carlo simulations. This technique allows for detailed alteration of the water and sediment discharge functions through hydromodification factors (k) as described in the succeeding texts.

Time series of random hydrologic events were simulated to characterize trends in sediment rating curves. First, instantaneous river discharge (Q) of each event was generated from random probabilities applied to an inverse lognormal cumulative distribution function that had a mean and standard deviation of $\ln(Q)$ of 4 and 0.6, respectively. Examples of 30 sequential discharge values generated by these techniques are shown in Figure 9a. Second, suspended-sediment discharge (Qs) for each event was generated by:

$$Qs = aQ^{b+1} + \varepsilon \qquad (6)$$

where a and b were set to 1 and 0.7, respectively, and ε is a random error term generated by an inverse normal cumulative distribution function (mean = 0; standard deviation = 1) scaled to 20% of each Qs estimate (Figure 9b). Lastly, suspended-sediment concentration was computed by the ratio of Qs and Q (Figure 9c). This technique resulted in a suspended-sediment rating curve consistent with power function formulations (Figure 9d). Although different distributions of Q, Qs and ε can be

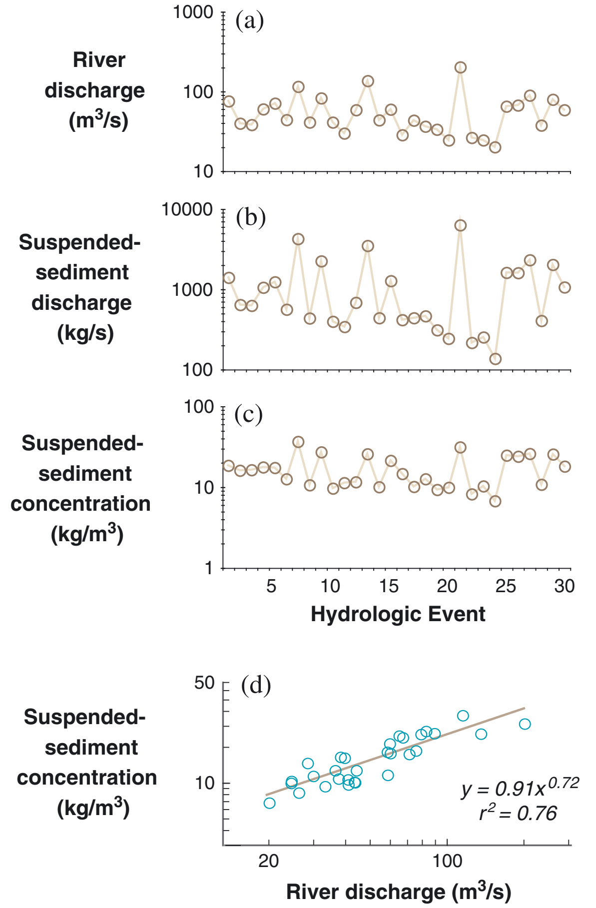

Figure 9. Records of (a) river discharge, (b) suspended-sediment discharge, and (c) suspended-sediment concentration generated by Monte Carlo simulations described in the text. (d) The power-function sediment rating curve from the data generated by Monte Carlo simulation

generated by modifying characteristics of the distributions described earlier, a complete review of these influences is beyond the scope of this paper. In the end, distributions were chosen that resulted in moderate linear correlation between $\log(Q)$ and $\log(C)$, for example, $r^2 = 0.76$ for the 30 events (Figure 9), so that the effects of change on the power-law formulations of Equation (3) could be assessed.

The first analysis using these techniques focused on two hydromodification scenarios presented earlier: (a) increases in suspended-sediment discharge and (b) increases in river discharge (Figure 10). These conditions mimic the simplified effects of wildfire and urbanization, respectively. That is, wildfire generally, but not universally, increases sediment supply to a greater degree than river discharge, and urbanization generally, but not universally, increases overland flow to a channel, which in the simplified case considered here results in negligible change in the river sediment supply, perhaps owing to channel armoring or another effect of urbanization. Field examples of these conditions can be found in Warrick *et al*. (2012) and Warrick and Rubin (2007), respectively.

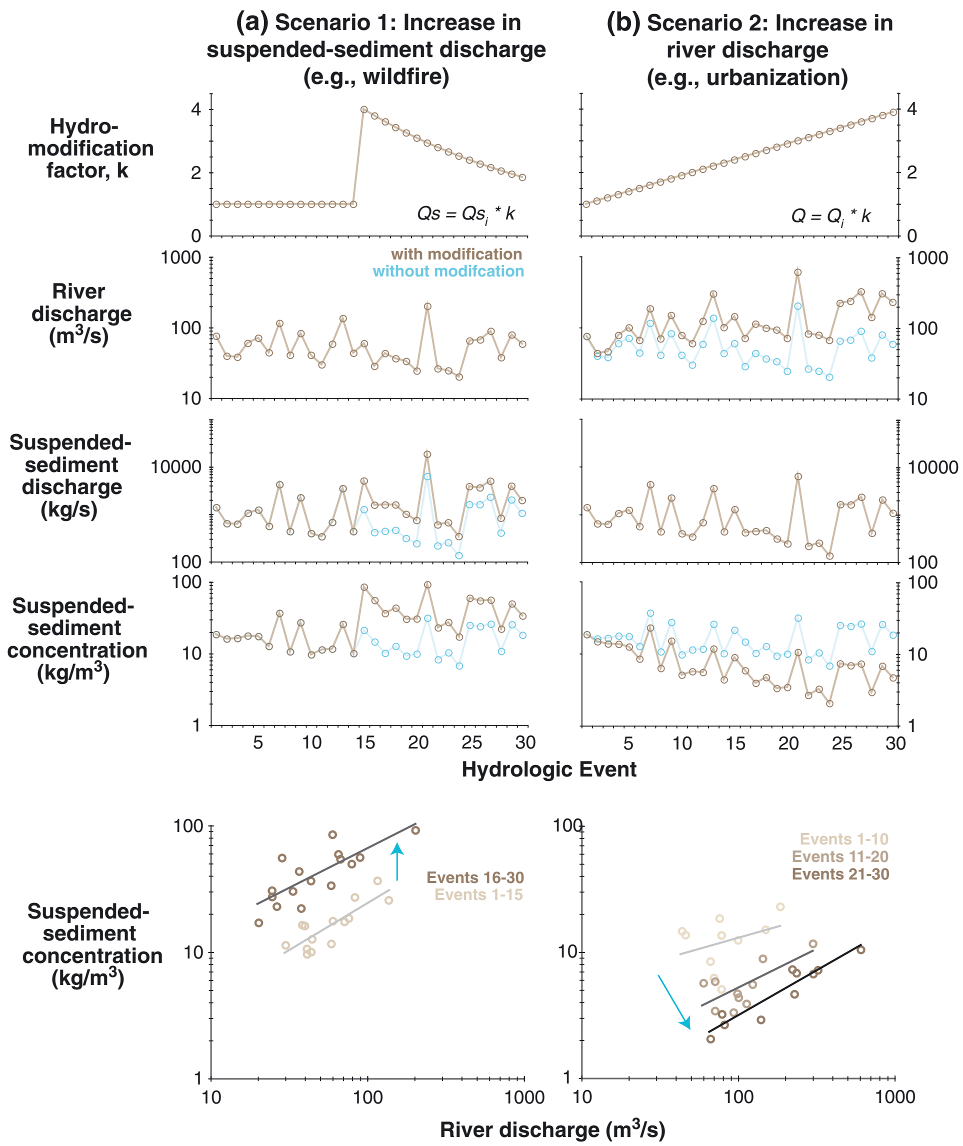

Figure 10. Simulations of river discharge, suspended-sediment discharge, suspended-sediment concentration, and the resulting suspended-sediment rating curves during 30 hydrologic events that include time-dependent hydromodifications. Modeled events are identical to those presented in Figure 9. (a) An increase in suspended-sediment discharge during events 16–30 that was initially fourfold and decayed 5% per event. (b) A linear increase in water discharge without modification of the suspended-sediment discharge. Power-function sediment rating curves for select events are shown with lines in the bottom panels

To model these changes, the data presented in Figures 9 were modified by the event-dependent k presented in the upper panels of Figure 10, which resulted in the modified hydrologic conditions presented in the middle panels of Figure (10).

The hydromodifications introduced for these two scenarios resulted in two different styles of sediment rating curve change. The sediment rating curve shifted upward in response to increases in sediment discharge but shifted downward in response to increases in water discharge (Figure 10, lower panels). For comparative purposes, the magnitude of the trend in vertical offsets has been tabulated as the ratios of the offset parameters (a and $\hat{a}$) computed using Equations (1) and (3) (Table I). The ratios in the fitted offsets were also compared with the theoretical vertical offset of the curves ($r_{\hat{a}}$), computed by:

$$r_{\hat{a}} = \frac{r_s}{(r_w)^{1+b}} \quad (7)$$

where r_s is the ratio of sediment discharge for two intervals of time, and r_w is the ratio of water discharge for the same time intervals (after Equation (6) in Warrick and Rubin, 2007). The ratios of fitted $\hat{a}$ closely resemble the theoretical changes computed with $r_{\hat{a}}$ from Equation (7) (Table I). The ratios of a, in contrast, differ greatly from the theoretical

Table I. Measured and theoretical change in sediment rating curve parameters from Monte Carlo simulations of two scenarios shown in Figure 10

	Sediment rating curve equation[a]	Ratio of offsets (a_n/a_1)[a]	Ratio of discharge-weighted offsets ($\hat{a}_n/\hat{a}_1$)[b]	Theoretical vertical offset in rating curves[c]
Scenario 1: increase in suspended-sediment discharge				
Events 1–15	$C = 0.764\ Q^{0.752}$	1	1	1
Events 16–30	$C = 3.73\ Q^{0.622}$	4.9	2.9	2.8
Scenario 2: increase in river discharge				
Events 1–10	$C = 2.60\ Q^{0.348}$	1	1	1
Events 11–20	$C = 0.309\ Q^{0.610}$	0.12	0.42	0.42
Events 21–30	$C = 0.115\ Q^{0.715}$	0.044	0.25	0.23

[a] From linear regression with Equation (1), where a_1 is the offset parameter during the first grouping of events, and a_n is the offset parameter during the nth grouping of events.
[b] From linear regression with Equation (3), where $\hat{a}_1$ is the offset parameter during the first grouping of events, $\hat{a}_n$ is the offset parameter during the nth grouping of events, and Q_{GM} is 50.8 and 116 m^3/s for scenarios 1 and 2, respectively.
[c] From the theoretical vertical offset in rating curves computed with Equation (7).

vertical offsets of $r_{\hat{a}}$ (Table I). This is consistent with presentation and discussion of Equation (5) earlier that suggests that $\hat{a}$ provides better assessments of vertical offsets and changes than a.

A second and more comprehensive assessment of the effects of changes to sediment rating curves was conducted with similar Monte Carlo techniques. For these analyses, 40 hydrologic events were generated with the first half having no hydromodification events and the second half having uniform hydromodifications. To produce vertical offsets that were roughly equivalent in scale by both water and sediment hydromodifications, Equation (7) was used to determine r_s and r_w such that $r_{\hat{a}}$ was equal to unity. Under these conditions, for example, it was found that an r_s of 3 was equivalent to an r_w of 0.52. Thus, the k-values for the second half of the simulated events were chosen to be 0.33 and 3 for sediment discharge and 0.52 and 1.91 for river discharge. Although a vast range of potential hydromodifications exist and should – eventually – be tested, the simple conditions evaluated here provide results that are instructive and important to examine.

The matrix of sediment rating curves resulting from the combination of uniform modifications to sediment and river discharge is shown in Figure 11. In general, changes to sediment discharge result in simple vertical shifts in the rating curve (Figure 11b and h), whereas changes to river discharge result in diagonal shifts in the rating curve (Figure 11d and f), the latter of which are consistent with the observations of Warrick and Rubin (2007). Changes in both sediment and river discharge were observed to be additive, such that rating curves were either further separated (Figure 11a and i) or shifted in a manner that no change in vertical offset occurred (Figure 11c and g). This latter effect is intriguing because it suggests that significant hydromodifications may occur over time without alteration of the sediment rating curve parameters (Figure 11c and g). Similarly, it is important to note that similar changes in rating curves can occur from very different hydromodifications in the river (e.g. compare Figure 11f and h). This provides evidence that changes in sediment rating curves are not diagnostic of unique hydromodifications of a river system.

Last, readers may have noticed that the simple scenarios considered in the aforementioned analyses did not result in trends in the slope parameter, b. Changes in b are easy to generate using these Monte Carlo techniques if k is dependent on Q or Qs. For example, a common characteristic of a urbanizing river basin is a nonlinear response in runoff, such that there are greater increases in the river discharge of frequent, smaller discharge events than the infrequent, larger discharge events (Leopold, 1968; Hollis, 1975). This effect can be modelled with a discharge-dependent k (Figure 12a), which results in an increase in the rating curve slope using the same 40 event simulation techniques described earlier (Figure 12b). In this example, nonlinear increases in discharge resulted in a downward vertical offset and an increasing slope in the rating curve over time (Figure 12b). Nonlinear hydromodifications are also possible in sediment discharge, and with some simple calculations, readers will find that these kinds of trends will similarly alter time series of $\hat{a}$ and b.

DISCUSSION

Computations of river sediment loads are important for understanding the geochemical, ecological and morphodynamic conditions and trends of fluvial and coastal systems. Sediment loads are especially important for understanding the status and trajectory of river mouth deltas

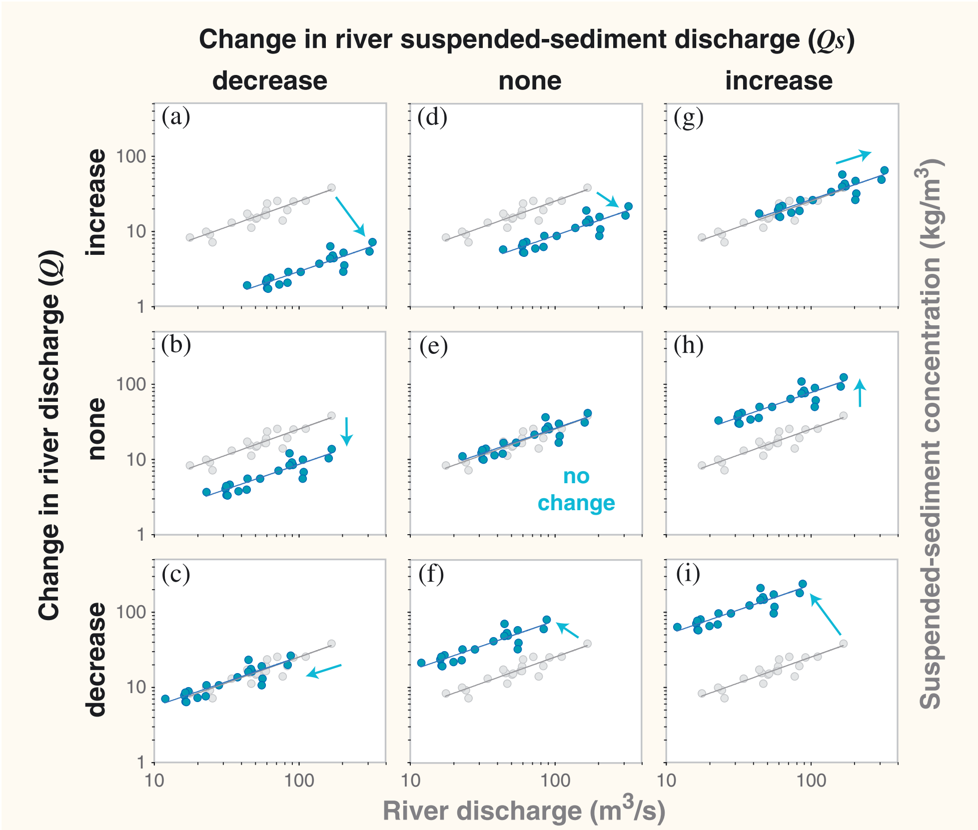

Figure 11. Time-dependent changes in suspended-sediment rating curves from modification of suspended-sediment discharge and/or river discharge. Results are based on Monte Carlo simulations of 40 hydrologic events that had hydromodification factors applied to the final 20 events (darker symbols). The hydromodification factors for sediment discharge were 0.33 (decrease) and 3 (increase), and the hydromodification factors for river discharge were 0.52 (decrease) and 1.91 (increase). These examples are only a small subset of the changes possible in river systems

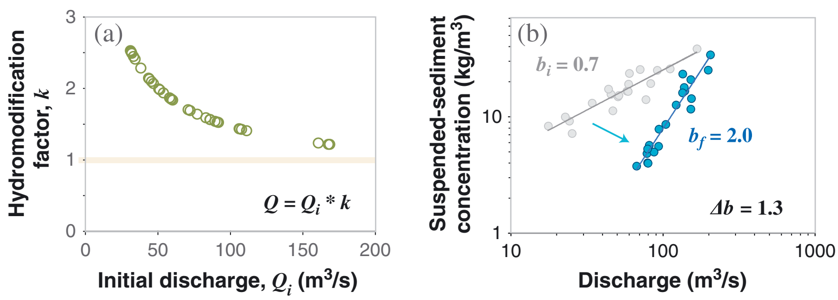

Figure 12. The effect of nonlinear increases in river discharge during steady sediment supply—such as may be experienced during urbanization–on the sediment rating curve parameters. Data used for the analyses are identical to those in Figure 11 except that the increase in river discharge for the final 20 events was discharge-dependent using a nonlinear hydromodification factor shown in (a) that had increases ranging between 1.2 and 2.5-fold. (b) The resulting changes to the sediment rating curve include an increase in the slope, *b*, from 0.7 to 2.0

and coastal sediment systems, which can undergo substantial morphologic and ecological change when sediment loads are altered (e.g. Yang *et al*., 2011; Warrick and Barnard, 2012). The adaptability of coastal wetlands under increased rates of sea level rise will likely be related to suspended-sediment loads and concentrations that provide material for these wetlands to accrete in response to sea level (Kirwan *et al*., 2010). Thus, it is important to adequately characterize changes in river suspended-sediment concentrations and discharge under the multitude of contributing watershed and climatic factors influencing these concentrations and loads (Milliman and Farnsworth, 2011).

For some river systems, the sediment supply relationships are adequately complex that rating curve techniques cannot describe patterns and trends, and other computational techniques are required to estimate sediment discharge (e.g. Aulenbach and Hooper, 2006; Wright *et al.*, 2010). However, for river systems that are more regularly behaved, sediment rating curves can be a useful – and perhaps powerful – tools for prediction (Asselman, 2000; Syvitski *et al.*, 2000; Horowitz, 2003). Although the use of sediment rating curves can be justified for these river systems, the results provided here suggest that care must be used when rating curves are used for trend analyses.

The use of the standard rating parameters, a and b, as derived from least squared regressions with Equation (1) is common in the hydrologic and geologic sciences, especially in analyses of hydrologic trends (e.g. Warrick and Rubin, 2007). However, these parameters may provide misleading results from the strong dependence of a on both b and Q. It is well shown that changes in a are strongly related to changes in b (e.g. Asselman, 2000; Syvitski *et al.*, 2000; Figure. 3, 4 and 5c), and Syvitski *et al.* (2000) report on the strong correlation between a and mean discharge in their global database. Thus, these dependencies should not be surprising. However, this suggests that a-values cannot be used as independent measurements of the vertical position of rating curves.

In lieu of Equations (1) and (2), the discharge-normalized formulations of Equations (3) and (4) are recommended for rating curve trend analyses. This is largely owing to the discharge-normalized equations providing better and independent measurements of the vertical offsets of rating curves. Normalization of discharge data is a common technique in hydrologic analysis, for example, to compare sediment rating curves across a set of watersheds (e.g. Hicks *et al.*, 2000). Normalization techniques are not likely to work well, however, if there is curvature in the relationship between log(Q) and log(C), as shown for the Eel River (Figures 7 and 8). This is consistent with the conclusions of Helsel and Hirsch (2002), who suggest that nonlinear relationships in log(Q) and log(C) data require analyses of residuals about a nonlinear fitting function, such as the lowess technique of Cleveland (1979). Here, it was shown that these nonlinear patterns result in discharge dependence of both $\hat{a}$ and b (Figure 8), which may confound trend analyses using power-law formulations.

It was also shown that trends in sediment rating curves could be caused by alterations of both the river discharge and sediment discharge regimes. Thus, rating curve changes are not diagnostic of unique changes in watersheds. For example, vertical shifts in sediment rating curves may be caused by changes in sediment supply, river discharge rates or both (Figure 11). It is therefore important to assess the time series of both river discharge and sediment discharge along with sediment rating curves. Precipitation can serve as an important independent variable to help assess these changes (e.g. Helsel and Hirsch, 2002).

Thus, sediment rating curves can be powerful, but not perfect, tools for assessing changes in rivers. Much of the strength and popularity of sediment rating curves is derived from their ease of use and simplistic graphical form (Figure 1). That noted, it is important to consider that Helsel and Hirsch (2002, p. 336) conclude that analyses of residuals about fitted functions (such as lowess and linear regressions) or parametric multiple regression (i.e. regressions that include time as a dependent variable) have more statistical power than a stage-wise regression analyses (such as time-dependent sediment rating curves). Thus, sediment rating curves do not provide the greatest statistical power for assessing time-dependent changes, and trend analyses for river suspended-sediment concentrations using other statistical techniques may provide better results. Readers may find that other techniques, such as analyses of residuals (e.g. Helsel and Hirsch, 2002), parametric multiple regression (e.g. Helsel and Hirsch, 2002), flow-weighted analyses of concentrations (e.g. Horowitz, 2009) or weighted-regressions (e.g. Hirsch *et al.*, 2010), better characterize the trends in suspended-sediment concentration and discharge than simple analyses by sediment rating curves.

CONCLUSIONS

Because of the broad popularity and utility of sediment rating curves, they will likely continue to be an important tool in the assessment and description of river sediment loads. Although sediment rating curves can provide important graphical and mathematical descriptions of river data and their trends with time, several limitations are inherent in their use. Two primary findings highlight these limitations. First, it was shown that the offset parameter, a, of the standard power-law sediment rating curve is a poor metric for the vertical offset and trends of a rating curve. Thus, new discharge-normalized formulations were developed and shown here to provide better results. Second, it was shown that trends in sediment rating curves were not diagnostic of unique river water or sediment discharge conditions. For these reasons, it is concluded that trend analyses with sediment rating curves must be accompanied with assessments of the time-dependent rates of river water and sediment discharge. It was also noted that other methods can provide better statistical power than sediment rating curves for trend analyses, and several suggestions were provided to accompany or replace rating curve techniques. Although the limitations of sediment rating curves do not preclude their use for trends analyses, they do suggest that care is needed in the interpretation of their results.

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